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MOND Kinematics of simple pendulum

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Abstract—Highlighting the behavior of a simple pendulum in view of the modified kinetic energy, due to Pankovic and Kapor in MOND theory by Milgrom, its Newtonian analogue is reclaimed.

Index Terms— Euler-Lagrange equation, Kinetic energy, Lagrangian, MOND, Potential energy, Simple pendulum.

1 Introduction

When the uniform velocity of rotation of cluster of galaxies was first observed (Oort 1932 and Zwicky 1933) it was not in tune with Newtonian theory of gravity. The galaxies or stars sufficiently away from the centre of the cluster move with constant velocities more than predicted by the theory. The most widely accepted approach to explain this problem postulates the existence of the dark matter. However, even after seven decades, there is no convincing evidence of the dark matter. In an attempt to explain the observed uniform velocities of galaxies without dark matter hypothesis, Professor Milgrom in 1983 propounded an equation of motion which resulted into a theory, known as MOND (Modified Newtonian Dynamics), which is a phenomenological scheme whose basic premise is that the visible matter distribution in a galaxy or cluster of galaxies alone determines its dynamics. In MOND the Newton's second law of motion is generalised as

$$\mathbf{F} = m\mu(a/a_0)\mathbf{a}\ , \tag{1.1}$$

Where μ is an interpolation function defined by

$$\mu(a/a_0) = \begin{cases} a/a_0 & a << a_0 \\ 1 & a >> a_0 \end{cases}.$$

For $a >> a_0$ above equation reduces to Newtonian one $\mathbf{F} = m\mathbf{a}$. The quantity a_0 is constant having the dimensions of acceleration $\mathbf{a}$ and is evaluated as $a_0 \approx 2\times 10^{-8} cm/s^2$ (Milgrom 1983, Bekenstein and Milgrom 1984). Even if MOND simply and elegantly describes important astronomical observational data, physical interpretation of the MOND is not simple at all. It is interesting that many of the researchers have directed their attention to probe into the various aspects of the MOND, from mathematical and physical point of views. Very recently, Pankovic and Kapor (2010) have concluded that MOND can be interpreted as a theory with the modified kinetic term of the conventional Newtonian dynamics. In this communication we go ahead with the implication of kinetic term with reference to the motion of a simple pendulum.

2. Simple pendulum in K-MOND

Consider a simple pendulum of mass m with the effective length l . Its kinetic energy in the Newtonian dynamics is

$$T = \frac{1}{2} m\, l^2\, \dot{\theta}^2\, ,$$

where θ be the angular displacement of the simple pendulum from the equilibrium position. Following Pankovic and Kapor (2010) we adopt the form of kinetic energy as

$$T = \frac{1}{2} m\, l^2\, \dot{\theta}^2 \left(\frac{l\,\ddot{\theta}}{l\,\ddot{\theta} + a_0} \right) \tag{2.1}$$

Then the Lagrangian L of the system has the form,

$$L = \frac{1}{2} m\, l^2\, \dot{\theta}^2 \left(\frac{l\,\ddot{\theta}}{l\,\ddot{\theta} + a_0} \right) - mgl(1-\cos\theta)$$

Since $L = L(\theta, \dot{\theta}, \ddot{\theta})$,it satisfies the generalized Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}} \right) - \frac{d^2}{dt^2}\left(\frac{\partial T}{\partial \ddot{\theta}} \right) = -\frac{\partial V}{\partial \theta} \tag{2.2}$$

We compute the left side of the above equation as follows,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}} \right) = \frac{ml^3\ddot{\theta}^2}{(l\ddot{\theta} + a_0)} + O_1 \tag{2.3}$$

and

$$\frac{d^2}{dt^2}\left(\frac{\partial T}{\partial \ddot{\theta}} \right) = \frac{ml^3 a_0 \ddot{\theta}^2}{(l\ddot{\theta} + a_0)^2} + O_2 \tag{2.4}$$

where the respective first terms at the right sides of (2.3), (2.4) are leading terms, while O_1 and O_2 represent non-leading terms whose forms are

$$O_1 = \frac{ml^3\dot{\theta}\theta^{(3)}}{(l\ddot{\theta}+a_0)^2}\left[a_0 - l\ddot{\theta}\right]$$

and

$$O_2 = \frac{ml^3 a_0\dot{\theta}}{(l\ddot{\theta}+a_0)^2}\left[\theta^{(3)} - \frac{4l\ddot{\theta}\theta^{(3)} + l\dot{\theta}\,\theta^{(4)}}{(l\ddot{\theta}+a_0)} + \frac{3l^2\dot{\theta}\,\theta^{(3)2}}{(l\ddot{\theta}+a_0)^2}\right]$$

Neglecting these approximate values, the Euler-Lagranges equation (2.2) assumes the

form $$\frac{ml^3\ddot{\theta}^2}{(l\ddot{\theta}+a_0)}\left[1 - \frac{a_0}{(l\ddot{\theta}+a_0)}\right] = -\frac{\partial V}{\partial \theta}. \qquad (2.5)$$

Considering $l\ddot{\theta} >> a_0$ for Newtonian regime (2.5) turns out approximately to

$$ml^2\ddot{\theta} = -\frac{\partial V}{\partial \theta} \qquad (2.6)$$

Similarly $l\ddot{\theta} << a_0$ for MOND regime, (2.5) approximately yields

$$\frac{ml^3\ddot{\theta}^3}{a_0{}^2} = -\frac{\partial V}{\partial \theta}$$

which is different from MOND, see (2.7)

Suppose, however that $l\ddot{\theta}$ is initially very small, i.e much smaller than a_0. In this case (2.1) approximates to

$$T = \frac{1}{2}ml^3\,\frac{\dot{\theta}^2\ddot{\theta}}{a_0}.$$

And by using above Kinetic Energy, we obtain

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \frac{ml^3\ddot{\theta}^2}{a_0},$$

In the same case $\frac{d^2}{dt^2}\left(\frac{\partial T}{\partial \ddot{\theta}}\right)$ can be neglected and then (2.2) becomes

$$\frac{ml^3\ddot{\theta}^2}{a_0} = -\frac{\partial V}{\partial \theta}. \qquad (2.7)$$

For $l\ddot{\theta} >> a_0$, (2.1) comes out to be

$$T = \frac{1}{2}ml^2\,\dot{\theta}^2.$$

Using above expression for Kinetic Energy, we deduce that

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta},$$

and $$\frac{d^2}{dt^2}\left(\frac{\partial T}{\partial \ddot{\theta}}\right) = 0.$$

In this case (2.2) assumes the form

$$ml^2\ddot{\theta} = -\frac{\partial V}{\partial \theta}. \qquad (2.8)$$

Since potential energy is arbitrary, defining a new potential called as K-MOND potential V/l instead of V and restoring the same symbol V for it, we get Newtonian analogue of simple pendulum (2.8) as

$$ml\ddot{\theta} = -\frac{\partial V}{\partial \theta}.$$

Also, the K-MOND analogue of simple pendulum from (2.7) can be deduced as

$$\frac{ml^2\ddot{\theta}^2}{a_0} = -\frac{\partial V}{\partial \theta}.$$

Conclusion

We have demonstrated the K-MOND analogue and restored the Newtonian analogue for a simple pendulum.

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